Quiz 6
Chemical Engineering Thermodynamics
February 25, 2016

1)
P6.1. Express in terms of P, V, T, C_P, C_V, and their derivatives. Your answer may include absolute values of S if it is not a derivative constraint or within a derivative.

a. $(\partial H/\partial S)_V$

b. $(\partial H/\partial P)_V$

c. $(\partial G/\partial H)_P$

2)
P5.4. An ordinary vapor compression cycle is to be operated on methane to cool a chamber to 112 K. Heat is rejected to liquid ethylene at 160 K. The temperatures in the coils are 168 K and 100 K.

a. Write the relevant energy and entropy balances for the compression step.

b. Estimate the minimal work requirement (J/g) for the compressor

c. Estimate the coefficient of performance (COP) for this OVC cycle.

d. Estimate the COP by the Carnot guideline.

e. Estimate the minimal work requirement for the compressor

In answering first sketch the OVC cycle, then make a table of the streams noting the state of the streams where it is important. Then solve for the values of H for each stream.

4.19 kJ/(kg-K) = 1 BTU/(lb-F)

°C * 9/5 + 32 = °F

$$\mu_{JT} \equiv \left(\frac{\partial T}{\partial P}\right)_H \quad \alpha_P \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \quad \kappa_T \equiv \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T$$

$$(\partial S/\partial T)_V = C_V/T \quad C_P \equiv (\partial H/\partial T)_P.$$

Maxwell's Relations

$$dU = TdS - PdV \Rightarrow -(\partial P/\partial S)_V = (\partial T/\partial V)_S \tag{6.29}$$

$$dH = TdS + VdP \Rightarrow (\partial V/\partial S)_P = (\partial T/\partial P)_S \tag{6.30}$$

$$dA = -SdT - PdV \Rightarrow (\partial P/\partial T)_V = (\partial S/\partial V)_T \tag{6.31}$$

$$dG = -SdT + VdP \Rightarrow -(\partial V/\partial T)_P = (\partial S/\partial P)_T \tag{6.32}$$

$$\left(\frac{\partial x}{\partial y}\right)_F\left(\frac{\partial y}{\partial F}\right)_x\left(\frac{\partial F}{\partial x}\right)_y = -1 \tag{6.15}$$

Triple product rule.

$$\left(\frac{\partial x}{\partial y}\right)_F = \left(\frac{\partial x}{\partial z}\right)_F\left(\frac{\partial z}{\partial y}\right)_F \tag{6.16}$$

Chain rule interposing a variable.

$$\left(\frac{\partial F}{\partial w}\right)_z = \left(\frac{\partial F}{\partial x}\right)_y\left(\frac{\partial x}{\partial w}\right)_z + \left(\frac{\partial F}{\partial y}\right)_x\left(\frac{\partial y}{\partial w}\right)_z \tag{6.17}$$

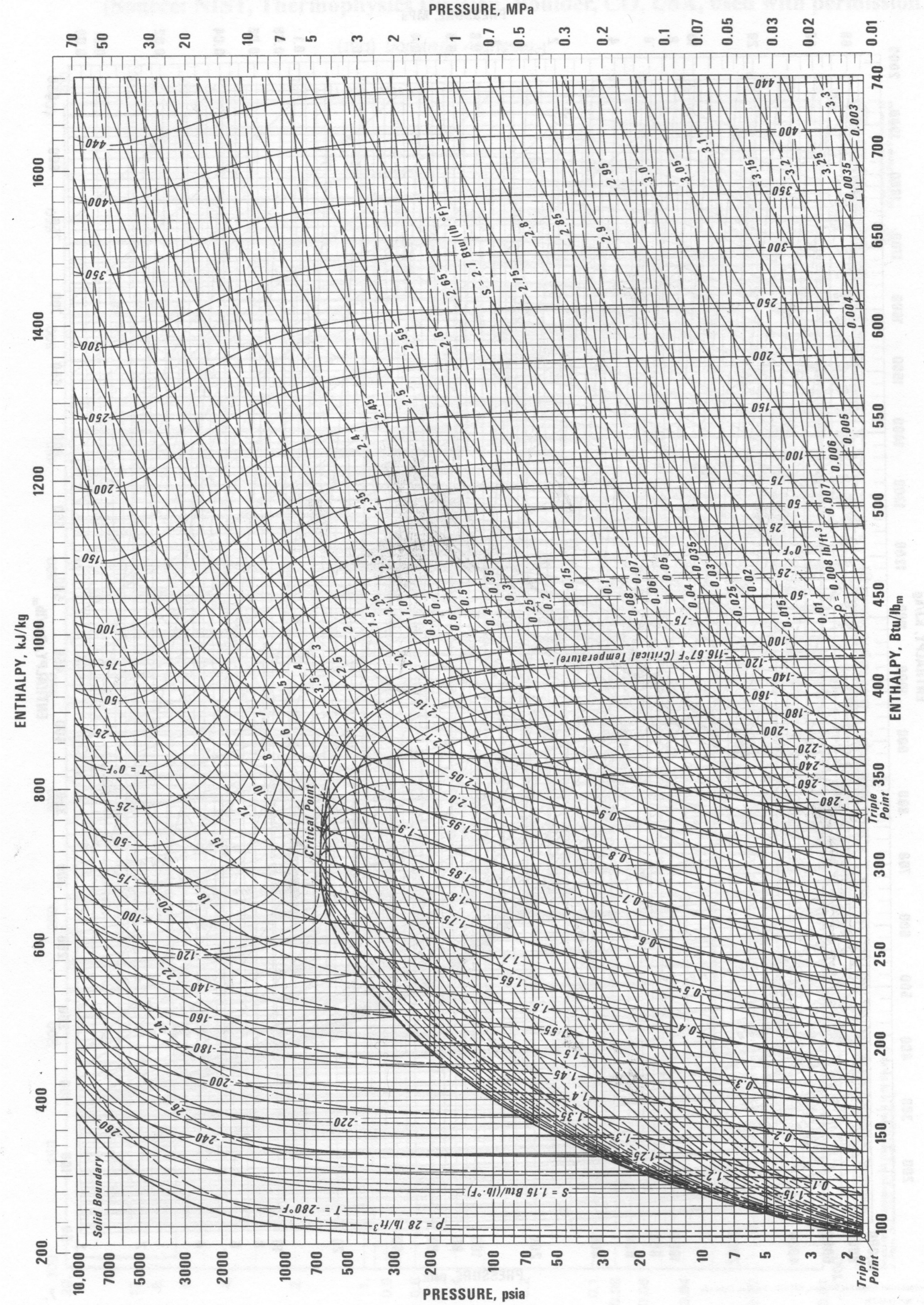
The expansion rule.

$$\left(\frac{\partial x}{\partial y}\right)_x = 0 \quad \text{and} \quad \left(\frac{\partial x}{\partial y}\right)_y = \infty$$

$$\left(\frac{\partial x}{\partial x}\right)_y = 1$$

E.10 PRESSURE-ENTHALPY DIAGRAM FOR METHANE

(Source: NIST, Thermophysics Division, Boulder, CO, USA, used with permission.)



Cryogenics Division, NBS-IBS, Boulder, Colorado
Chart 2029W (4.1.77)

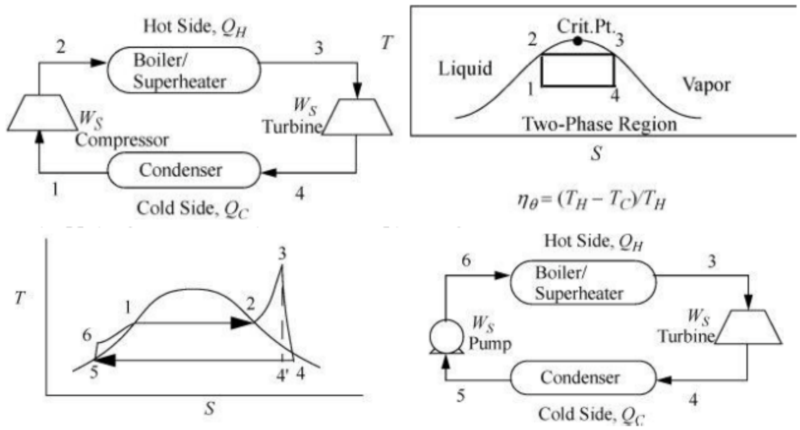


Figure 5.2. Rankine cycle.

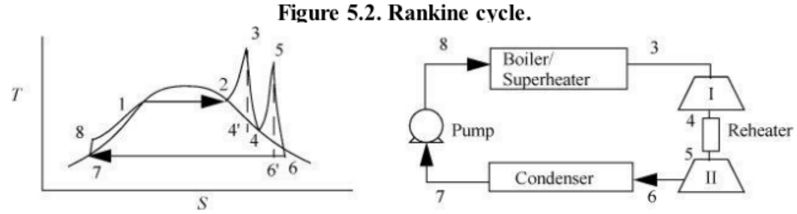


Figure 5.3. Rankine cycle with reheat.

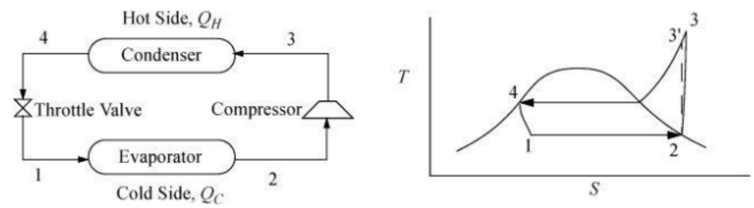


Figure 5.8. OVC refrigeration cycle process schematic and T-S diagram.

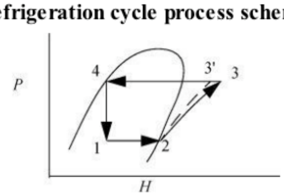


Figure 5.9. OVC refrigeration cycle plotted on the more commonly used P-H diagram. State numbers correspond to Fig. 5.8.

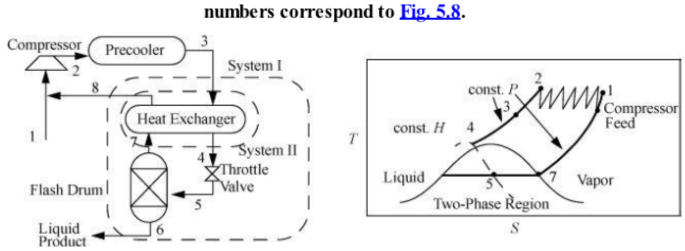


Figure 5.12. Linde liquefaction process schematic. The system boundaries shown on the left are used in Example 5.5.

ANSWERS
Quiz 6
Chemical Engineering Thermodynamics
February 25, 2016

1)

P6.1. Express in terms of P, V, T, C_P, C_V, and their derivatives. Your answer may include absolute values of S if it is not a derivative constraint or within a derivative.

a. $(\partial H/\partial S)_V$

b. $(\partial H/\partial P)_V$

c. $(\partial G/\partial H)_P$

(P6.1)

(a)

$$\left(\frac{\partial H}{\partial S}\right)_V$$

Expansion Rule: $dH = TdS + VdP$

$$\left(\frac{\partial H}{\partial S}\right)_V = T\left(\frac{\partial S}{\partial S}\right)_V + V\left(\frac{\partial P}{\partial S}\right)_V$$

$$= T + V\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial T}{\partial S}\right)_V \quad \text{(chain rule)}$$

$$= T\left(1 + \frac{V}{C_V}\left(\frac{\partial P}{\partial T}\right)_V\right)$$

(b)

$$\left(\frac{\partial H}{\partial P}\right)_V$$

Expansion rule: $dH = TdS + VdP$

$$\left(\frac{dH}{\partial P}\right)_V = T\left(\frac{\partial S}{\partial P}\right)_V + V\left(\frac{\partial P}{\partial P}\right)_V$$

$$= T\left(\frac{\partial S}{\partial T}\right)_V\left(\frac{\partial T}{\partial P}\right)_V + V = C_V\left(\frac{\partial T}{\partial P}\right)_V + V$$

(c)

$$\left(\frac{\partial G}{\partial H}\right)_P$$

chain using T since it is measureable

$$\left(\frac{\partial G}{\partial T}\right)_P\left(\frac{\partial T}{\partial H}\right)_P = \left(\frac{\partial G}{\partial T}\right)_P / \left(\frac{\partial H}{\partial T}\right)_P = \frac{\left(\frac{\partial G}{\partial T}\right)_P}{C_P}$$

Use expansion rule: $dG = -SdT + VdP$

$$\left(\frac{\partial G}{\partial T}\right)_P = -S\left(\frac{\partial T}{\partial T}\right)_P + V\left(\frac{\partial P}{\partial T}\right)_P = -S$$

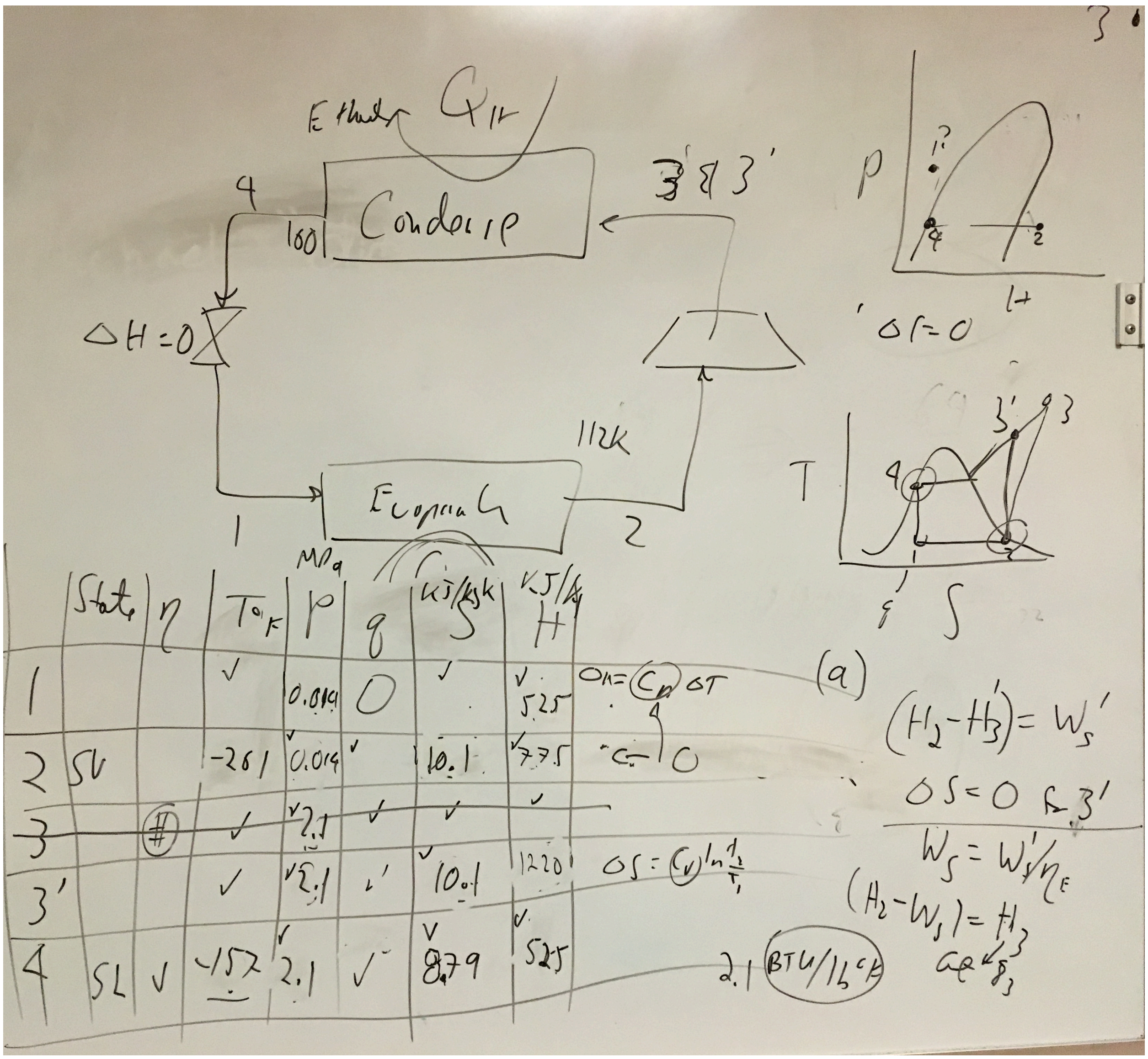
thus:

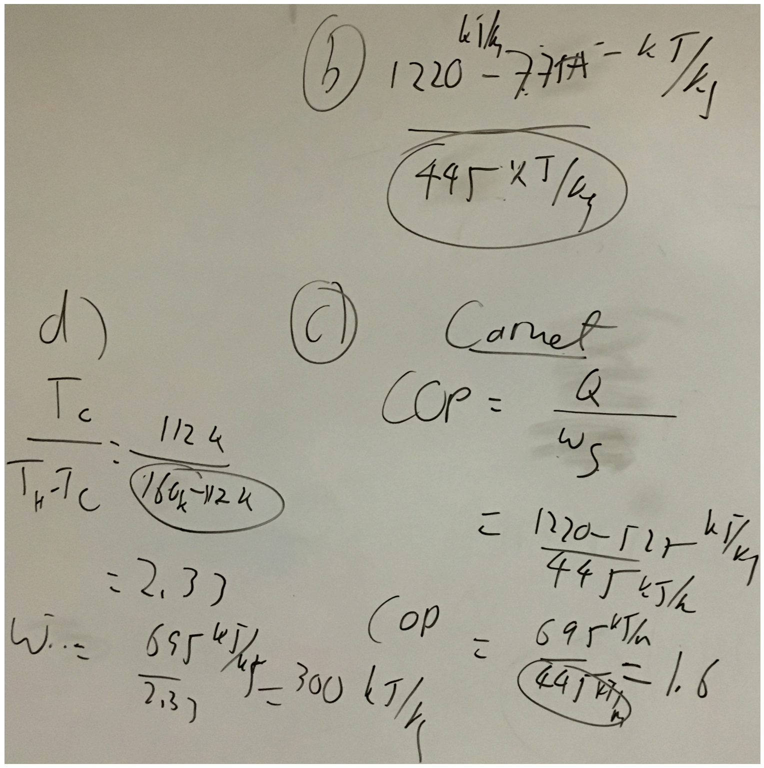
$$\left(\frac{\partial G}{\partial H}\right)_P = -\frac{S}{C_P}$$

2)

P5.4. An ordinary vapor compression cycle is to be operated on methane to cool a chamber to 112 K. Heat is rejected to liquid ethylene at 160 K. The temperatures in the coils are 168 K and 100 K.

- **a.** Write the relevant energy and entropy balances for the compression step.
- **b.** Estimate the minimal work requirement (J/g) for the compressor
- **c.** Estimate the coefficient of performance (COP) for this OVC cycle.
- **d.** Estimate the COP by the Carnot guideline.
- **e.** Estimate the minimal work requirement for the compressor




b) 1220 kJ/kg − 775 kJ/kg
445 kJ/kg
d)
Tc / (TH − Tc) = 112 K / (160 K − 112 K)
= 2.33
W = 695 kJ/kg / 2.33 = 300 kJ/kg
c) Carnot
COP = Q / Ws
= (1220 − 525 kJ/kg) / 445 kJ/kg
COP = 695 kJ/kg / 445 kJ/kg = 1.6